Use of Lagrangian Statistics to Describe Slurry Transport

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Lagrangian statistics are explored as a means of describing the transport of solids in a turbulently flowing liquid. The essential feature of the approach is to represent the concentration field as resulting from a distribution of sources of particles. It is argued that this provides a better framework to understand the physics than the Eulerian analysis currently being used. Fully developed concentration fields are calculated, using the assumptions of homogeneous turbulence and plug flow. It is found that the configuration of solids and the suspended load depend primarily on the ratio of the settling velocity to the friction velocity, which is a measure of the relative importance of turbulence and of settling in depositing particles. The analysis emphasizes the need for a better understanding of the mechanism and the rate of entrainment of particles into a turbulently flowing liquid.

Introduction

The flow of a slurry of solid particles in a horizontal open or closed channel has received considerable attention. Because of gravitational effects, the solids distribute asymmetrically in the liquid and settle out on the bottom of the channel. If the liquid velocity is large, the rate which settled particles are removed from the wall is such that complete suspension occurs. At low liquid velocities, however, the rate of removal is smaller and a bed forms on the bottom wall.

This article explores the use of Lagrangian statistics to describe the concentration profile of suspended sediment and the load carried by the liquid when a bed is present. The concentration is considered small enough that the fluid turbulence is not affected by the presence of the particles and that particle-particle interactions are not important. The bottom of the channel supplies particles to the system at a rate per unit area defined as R_A, and particles deposit at a rate defined as R_D. The approach taken is to consider the concentration field as resulting from a distribution of sources along the sediment bed, which have a strength proportional to R_A.

The principal theoretical problem, then, is to describe the behavior of one of these sources. A difficulty is that the fluid turbulence and mean velocity fields are nonhomogeneous. One approach is to use a random flight method of the type developed by van Dop et al. (1985) and Durban (1980) for an inhomogeneous turbulence. There are uncertainties in how to account for nonhomogeneities in the modified Langevin equations used by these authors. Furthermore, the need to carry out numerical calculations for a large number of particle trajectories makes it cumbersome to explore a wide range of flow conditions. Consequently, the simplifying assumptions of a homogeneous turbulence and of a homogeneous mean flow field are made. This allows for the use of Taylor's (1921) theory for diffusion from a small source in a homogeneous turbulent field. Future work will require a consideration of how nonhomogeneities in the flow field affect the results presented in this article.

The influence of gravity on dispersion is taken into account by recognizing that it acts independent of the turbulence. Particles entering the flow field are influenced by a constant force, $m_p g$, that has the effect of defining an average deterministic trajectory. The influence of turbulence and its contributions to diffusion are reflected in the deviations of actual trajectories from the mean, in defining boundary conditions and in specifying how particles enter the field. Since the Lagrangian correlation is assumed to be an exponential function, the approach is essentially the same as using the Langevin equation with an additional acceleration associated with the difference of the gravitational and mean drag forces.

The inertia of the particles is characterized by a reciprocal time constant:

$$\beta=\tau_P^{-1}=\frac{3f_D\rho}{4d_P\rho_P}\,|Q_R|, \tag{1}$$

where f_D is the drag coefficient, ρ, the fluid density, ρ_P, the particle density, d_P, the particle diameter, and $|Q_R|$, the absolute value of the relative velocity between the fluid and the particle. The ratio of the time scale of the turbulence to the inertial time scale of the particle is represented by $\beta\tau_{LF}$, where τ_{LF} is the Lagrangian time scale of the fluid. For small $\beta\tau_{LF}$, the particles do not follow the turbulence, so the turbulence properties of the particles differ from the turbulence characteristics of the fluid.

The inertial time constant of the particle made dimensionless with wall parameters, $\tau_P^+ = v^{*2}\tau_P/\nu$, is a measure of the ratio of the stopping distance of a particle in a still fluid to the thickness of the viscous wall region. For $\tau_P^+ > 20$ (McCoy and Hanratty, 1975), the stopping distance is larger than the viscous wall region, so the detailed flow properties in this region are not important. (Aerosol particles are characteristics by $\tau_P^+ < 20$; as a consequence, their deposition is influenced strongly by nonhomogeneities close to the wall.)

Another parameter which is important in relating turbulence characteristics of the particles to those of the fluid is the ratio of the settling velocity of the particles in a stationary fluid, V_T, to the magnitude of the turbulent velocity fluctuations. A measure of this ratio is V_T/v^*.

Recent experimental studies (Young and Hanratty, 1991a,b; Lee et al., 1989a,b) and theoretical (Reeks, 1977; Pismen and Nir, 1978; Nir and Pismen, 1979) studies provide a means of relating turbulent characteristics of the particles to those of the fluid through the dimensionless groups $\beta\tau_{LF}$ and V_T/v^*. For very fine particles (very large $\beta\tau_{LF}$, very small V_T/v^* and τ_P^+), the suspension is uniform, and the principal problem seems to be associated with the influence of the particles on the rheological properties of the flow (Govier and Aziz, 1972). The analysis in this article considers nonhomogeneous suspensions in a liquid for which $1 < \beta\tau_{LF} < 100$, $V_T/v^* \leq 1$ and $\tau_P^+ > 20$. For this range of parameters, turbulence characteristics of the particles are approximately the same as those of the fluid. Furthermore, τ_P^+ is large enough that the particles move in free flight through the viscous wall region.

The limitation of the analysis to large τ_P^+ provides the justification for the use of a homogeneous flow model (and Taylor's theory of diffusion) as a first-order approximation. The largest flow nonhomogeneities occur in the viscous wall region. Since the particles move to the wall in free flight, the influence of turbulence on depositing particles can be ignored in this region.

This work has a kinship to an analysis by Hunt and Nalpanis (1985) which represents concentration profiles in the saltation regime as the result of a series of deterministic particle trajectories that originate from the bed surface. In a sense, it extends this analysis to include the effect of nondeterministic effects of turbulent diffusion.

Lagrangian Description of a Wall Source

Description of a wall source

The system considered is a horizontal rectangular channel in which fluid is flowing in the x direction. The bottom boundary, a bed of sediment, is at $y=0$ and the top boundary, at $y=H$. Diffusion in the x direction is neglected. The fluid velocity field is considered to be fully developed. Because of the assumption of uniform flow, changes in the x direction can be considered as changes in time since

$$dt = \frac{dx}{U} \tag{2}$$

The concentration field is described as resulting from sources in the fluid at $x=0$ (or $t=0$) and at the bottom wall for $x \geq 0$ ($t \geq 0$).

Particles are removed from the bottom boundary by the liquid flow. These particles enter the field with a distribution of velocities from a line source at time t'. The averages of the concentration field and the redeposition rate are sought at time $(t-t')$ for a large number of particles. Since the problem is linear, the analysis can be carried out separately for each particle size and for each entering velocity. To simplify the calculations, the particles are assumed to have one size. Because of a lack of knowledge about how particles are entrained, the assumption is made that all the particles enter the field at a single velocity, with components in the x and y directions of U_o and V_o.

The motion of the particles is affected by fluid turbulence and by the gravitational field. Typical paths taken by the particles are indicated in Figure 1. The velocity components along a trajectory may be considered as the sum of the deterministic average path, defined by $U(t-t')$ and $V(t-t')$, and a random turbulent motion.

Velocity component $V(t-t')$ is given by the following equation:

$$m_P \frac{dV}{dt} = -\frac{1}{2}\rho f_D A_P |Q_R| V_R - g m_P \left(1 - \frac{\rho}{\rho_P}\right) \tag{3}$$

Here, $Q_R^2 = U_R^2 + V_R^2$; U_R is the x component of the relative velocity between the solid and the fluid; $V_R = V$, since there is no net fluid motion in the y direction; A_p is the projected area of the particle; and m_p is the particle mass. Velocity component U is defined by the following equation:

$$m_P \frac{dU}{dt} = -\frac{1}{2}\rho f_D |Q_R| U_R \tag{4}$$

Equations 3 and 4 are solved with the velocity at $t=t'$ specified by U_o and V_o. For $t > t'$ average velocity V will be less than V_o. For t much larger than t', there will be an average drift toward the wall given by $V = -V_T$.

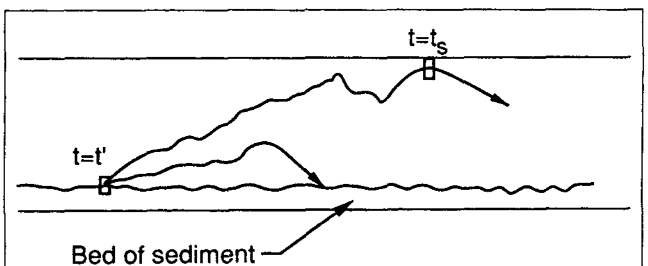

Figure 1. Sediment transport in a horizontal channel; typical trajectories from a wall source.

Thus, the particles in the channel can have a range of average velocities from $+V_o$ to $-V_T$. It is difficult to account for this in a Eulerian framework. This, however, is not a problem in a Lagrangian analysis, since all the particles originating from an infinitesimal wall source with velocity V_o will have the same velocity at any fixed value of $t-t'$.

The average contribution of $V(t-t')$ to the mixing of particles, from a single source, in the y direction can be described by the equation:

$$\frac{\partial C}{\partial t} = -V(t-t')\frac{\partial C}{\partial y}, \tag{5}$$

where it should be noted that, for a single source, V is a function of $(t-t')$ and not of y. The x component of the deterministic trajectory, $U(t-t')$, affects the calculation of $C(y, t-t')$ only through the calculation of Q_R, that is, in the evaluation of $f_D|Q_R|$ in Eq. 3. For the case of Stokesian drag, $f_D|Q_R|$ is constant and U need not be considered.

The turbulent components of the particle velocity will be designated by u and v. Because of the assumption of a homogeneous stationary field, $(\overline{v^2})$ and $(\overline{u^2})$ are constants. This assumption of stationarity need not be made. Then, if it is considered, however, the turbulent components at time t', u_o, and v_o, need to be specified. No information on u_o and v_o is presently available, so the additional complications associated with nonstationarity were not addressed.

Consequently, it is assumed that the particles are fully entrained in the turbulence a very short time after their entry into the field, so that the $V_o = (\overline{v^2})^{1/2}$ is taken as the initial condition for Eq. 3. Velocity U_o may be taken as S_oU^F, where U^F is the fluid velocity and S_o is the slip ratio of the particles entering the field. Equation 4 may be looked upon as defining the variation of the slip ratio, S, with time. Again, for simplification, S is taken as unity. As a consequence, $U_R = 0$ and $|Q_R| = |V|$.

If the distribution function describing v is Gaussian, the average turbulent component of the turbulent velocity with which particles strike the boundary is $\sqrt{(2/\pi)(\overline{v^2})}$. The rate of deposition to the bottom wall may be considered as the sum of contributions due to turbulence and gravitational settling:

$$R_{DO} = R_{DO}^T + R_{DO}^S \tag{6a}$$

The turbulence contribution was formulated to be consistent with arguments presented by Lee et al. (1989a,b) in their interpretation of laboratory studies of dispersion of droplets from a point source in a vertical flow ($V=0$):

$$R_{DO}^T = (2/\pi)^{1/2}(\overline{v^2})^{1/2} f(t-t')C(y=0, t-t') \tag{6b}$$

The function f is the fraction of particles at $y=0$ moving toward the wall. For small $(t-t')$, $f=0$, since all of the particles are entrained in turbulent motions away from the wall at $t=t'$. For large $(t-t')$, turbulent velocity fluctuations are just as likely in the plus as in the minus direction, so $f=1/2$.

The contribution due to settling in Eq. 6a is:

$$R_{DO}^S(t-t') = -V(t-t')C(y=0, t-t') \tag{6c}$$

For small $(t-t')$, $R_{DO}^T \cong 0$ and R_{DO}^S is negative, so Eq. 6a gives a negative value for the deposition rate. In these cases, R_{DO} is taken as zero.

The rate of impaction on the top boundary is given as:

$$R_{DH}(t-t') = [(2/\pi)^{1/2}(\overline{v^2})^{1/2} f(t-t') + V(t-t')]C(y=H, t-t') \tag{7}$$

where it is noted that the contribution from gravity has a different sign than in Eq. 6. For the situations to be considered $f(t-t') \approx 1/2$, since the particles reaching $y=H$ have been in the field for a long time. For situations in which the term in the brackets is negative, R_{DH} is zero since $C(y=H, t-t')$ would then be close to zero.

Particles that reach the top boundary at time t_S are pictured to be reentrained immediately: that is, they form new sources. If the deterministic velocity, on arrival, has a negative value, the velocity, V_H, with which particles leave the top boundary is:

$$V_H(t_S-t') = -V_o + V(t_S-t') \tag{8a}$$

Here, V_o is velocity with which the turbulent fluid entrains the particles; it is taken to be the same as for entrainment at the bottom surface. If $V(t_S-t')$, at arrival, is positive, then it assumed that the particle is stopped by the wall so that

$$V_H(t_S-t') = -V_o \tag{8b}$$

That is, the particles are assumed to roll along the surface and not to bounce. Condition 8a represents all of the calculations presented here.

The neglect of bouncing suggests that the upper boundary is completely absorbing. However, the assumption that there is no delay time before reentry is a feature of a bouncing particle. Clearly, other assumptions for the interaction of particles with the upper boundary would be reasonable. The model described above was used, because it gives the correct behavior in the limits of small and large gravitational effects.

Equations defining a line source

Taylor (1921) showed that turbulent dispersion of particles from a point or line source in a homogeneous field of infinite extent can be represented by the mean-squared displacement of particles from the origin. An effective turbulent diffusivity $\epsilon = 1/2(d\overline{Y^2}/dt)$ can be defined analogous to Einstein's definition of molecular diffusion as $\overline{Y^2} = 2D(t-t')$. The two processes differ in that ϵ $(t-t')$ depends on time and is constant only for large $(t-t')$.

Batchelor (1949) developed the following relation to represent the concentration field resulting from a distribution of sources in a homogeneous isotropic turbulence of infinite extent:

$$\frac{\partial C}{\partial t} = \frac{1}{2}\frac{d\overline{Y^2}}{dt}\nabla^2 C + S\delta(\vec{x}/\vec{x}')\delta(t/t') \tag{9}$$

Here, S is the source strength at $\vec{x}'$, t, having the units of mass per unit area, and the δ's are delta functions having the units of reciprocal length and reciprocal time.

Equation 9 resembles the Eulerian mass-balance equation and implies that a local flux is described by a gradient model:

$$N_i = -\epsilon(t-t')\frac{\partial C}{\partial x_i} \tag{10}$$

However, one should not conclude that a gradient model like Eq. 9 can be applied to a field which is the result of a number of sources and sinks. It is possible to calculate such a field from a knowledge of point source dispersion. However, it is not possible to do the reverse calculation.

Equation 9 has been used to describe the diffusion of heat from line or ring sources at a wall into a field of limited extent (Hanratty, 1956, 1958; Hanratty and Flint, 1958; Eckelman and Hanratty, 1972). In these analyses, Eq. 9 was solved for the behavior of a single wall source by using a zero flux boundary condition, $\partial C/\partial y = 0$ or $\partial C/\partial r = 0$, at the wall for $t > t'$.

Binder and Hanratty (1991) used Eq. 9 to describe droplet diffusion in vertical gas-liquid annular flow. Here, droplets are removed from the wall layer by the high-speed gas flow and eventually redeposited on the wall layer. Equation 9 was solved for a ring source at the wall using the assumption of a completely absorbing boundary. At the pipe wall the turbulent flux was set equal to the diffusive flux for $t > t'$. For the case of a vertical channel flow, this would be formulated as:

$$(2/\pi)^{1/2}(\overline{v^2})^{1/2} f(t-t')C = -\epsilon(t-t')\frac{\partial C}{\partial y} \tag{11}$$

at $y=0$, $t>t'$ (Lee et al., 1989a,b; Binder and Hanratty, 1991).

The turbulent diffusivity, ϵ, may be considered as the product of a velocity term, U, and a length term, L, where U is of the order of $(\overline{v^2})^{1/2}$. Therefore, from Eq. 11,

$$\mathsf{U}C \sim \mathsf{U}L\frac{\partial C}{\partial y} \tag{12}$$

For the case of molecular transport L is small. Since $\partial C/\partial y$ is finite, C is zero at a completely absorbing boundary. For the case of particle transport, L is large, so a completely absorbing boundary will be compatible with a finite concentration at $y=0$, that is defined by Eq. 11.

This article uses our earlier formulation (Binder and Hanratty, 1992) for horizontal gas-liquid annular flow by picturing the combined effect of gravity and turbulence on the behavior of a single line source on the wall as a combination of Eqs. 5 and 9. Thus, by neglecting diffusion in the x direction,

$$\frac{\partial C}{\partial t} = \epsilon(t-t')\frac{\partial^2 C}{\partial y^2} - V(t-t')\frac{\partial C}{\partial y} + S\delta(y/o)\delta(t/t') \tag{13}$$

From the discussion associated with Eqs. 6 and 13, concentration gradients at the boundaries are defined in terms of the turbulence contribution to the deposition flux. Therefore, the boundary conditions for Eq. 13 for sources on the bottom boundary are:

$$-\epsilon(t-t')\frac{\partial C}{\partial y} = \left(\frac{2}{\pi}\right)^{1/2}(\overline{v^2})^{1/2} C f(t-t') \tag{14}$$

at $y=0$, $t>t'$ and

$$\epsilon(t-t')\frac{\partial C}{\partial y} = \left(\frac{2}{\pi}\right)^{1/2}(\overline{v^2})^{1/2} C f(t-t') \tag{15}$$

at $y=H$, $t>t'$. The behavior of secondary sources formed on the top boundary at t_S are described by Eqs. 13, 14, and 15 with t_S substituted for t'. The chief difference from sources on the bottom wall is that $V(t-t_S)$ will be calculated from Eq. 3 with V at $t=t_S$ given by Eq. 8a.

An understanding of Eqs. 13, 14, and 15 can be obtained by recognizing that the left side of Eq. 13 represents the concentration change seen by an observer moving with velocity $V(t-t')$. Thus, if y is replaced by $\eta = y - \int V dt$, Eq. 13 takes the same form as Eq. 9:

$$\frac{\partial C}{\partial t} = \epsilon(t-t')\frac{\partial^2 C}{\partial \eta^2} + S\delta(\eta|o)\delta(t/t') \tag{16}$$

The boundary conditions were discussed fully by Binder and Hanratty (1992), who integrated Eq. 13 with respect to time, from t' to ∞, and with respect to y, from 0 to H. In this way, it is shown that Eqs. 14 and 15 are required to satisfy conservation of mass.

Definition of $\epsilon(t-t')$ and $f(t-t')$

The solution of Eqs. 13, 14 and 15 requires the definition of $\epsilon(t-t')$ and $f(t-t')$. According to Taylor's theory, the mean-squared displacement of particles released from a line source, at time zero in a homogeneous turbulent field, is defined by the equation:

$$\frac{1}{2}\frac{d\overline{Y^2}}{dt} = \overline{v^2}\int R_L(t)dt \tag{17}$$

where $\overline{v^2}$ is the mean square of the y component of the velocity fluctuations, and $R_L(t)$ is the Lagrangian correlation defined by:

$$R_L(t) = \frac{\overline{v(o)v(t)}}{\overline{v^2}}. \tag{18}$$

The numerator is the average product of the velocities of a particle at times zero and t. For small times, $v(o) \cong v(t)$ so that $\overline{v(o)v(t)} = \overline{v^2}$ and $R_L(t) \to 1$. For large times, $v(t)$ is not related to $v(o)$ so that $v(o)v(t)$ can be plus as often as it is minus. Consequently, $\overline{v(o)v(t)} = 0$ and $R_L \to 0$ for $t \to \infty$. The integral in Eq. 17 for $t \to \infty$ is a constant which is defined as the Lagrangian time scale of the fluid:

$$\tau_{LF} = \int_o^\infty R_L(t)dt \tag{19}$$

Since the Lagrangian turbulent diffusion coefficient is defined as:

$$\epsilon(t) = \frac{1}{2}\frac{d\overline{Y^2}}{dt} \tag{20}$$

it follows from Eqs. 17 and 18 that

$$\epsilon(t) = \overline{v^2} t \tag{21}$$

for $t \to 0$ and that

$$\epsilon = \overline{v^2} \tau_{LF} \tag{22}$$

for $t \to \infty$. The turbulent diffusion coefficient is, therefore, time-dependent. It varies linearly with time for small time and is a constant for large time. For the case in which the correlation coefficient is an exponential function,

$$R_L = \exp(-t/\tau_{LF}) \tag{23}$$

this time dependency is given as:

$$\epsilon = \overline{v^2} \tau_{LF} (1 - e^{-t/\tau_{LF}}) \tag{24}$$

The function $f(t-t')$ is assumed to be of the form:

$$f(t-t') = \frac{1}{2} \left[1 - \exp \frac{-(t-t')}{\tau_{LF}} \right] \tag{25}$$

This gives $f(0) = 0$ and $f(\infty) = 1/2$. The choice of this function was made for mathematical convenience, although it is physically reasonable since it would not be expected that $f(t-t')$ equals 1/2 until the dispersing particles have obtained the haphazard motion characteristic of diffusion at large times.

Eulerian Analysis

Before presenting results of calculations based on the Lagrangian analysis, it is appropriate to outline the Eulerian formulation. The main theory presently used to describe sediment transport argues that the concentration profile results from a balance between the settling of particles and turbulent diffusion (O'Brien, 1933; Rouse, 1937). For a fully developed concentration field, this is represented by:

$$E_P(y) \frac{dC}{dy} - VC = 0 \tag{26}$$

Here, $E_P(y)$ is the diffusion coefficient defined from an Eulerian framework; it is usually set equal to or proportional to the eddy viscosity of the fluid. Term V is the average particle velocity in the y direction; it is taken as the free fall velocity of a particle in a stationary fluid, $-V_T$. Sometimes it is argued that the absolute value of V will be less than V_T since the effective viscosity of the fluid increases with increasing particle concentration. However, a consideration which seems to be overlooked by most authors is that the particles might have velocities different from $-V_T$ because they have not been in the fluid long enough to reach their free-fall velocity.

The integration of Eq. 2 gives an infinite value of C at $y = 0$. Hunt (1969) has suggested that this difficulty can be removed if it is argued that the concentration close to the lower boundary is large enough that gradients in particle concentration are associated with a flux of fluid away from the surface as well as a flux of particles toward the surface. However, usual practice is to specify the concentration at some finite distance from the boundary.

Equation 26 suggests that the only mechanism for particles to deposit is gravitational settling. This is a concern because the deposition of particles which are not too large should be associated both with diffusion and gravitational effects. Sometimes, on the basis of Eq. 26, deposition rates are calculated as the product of the settling velocity and a measured concentration close to the bottom. Even for very large particles, this calculation could be in error because not all the particles would be moving toward the bottom. (Those being entrained in the fluid would actually be moving away from the bottom surface.)

Equation 26 is derived for an equilibrium situation in which there is no net transfer of particles between the liquid phase and the particle bed moving along the bottom wall. Its extension to a developing flow has been discussed in a number of articles (Hjelmfelt and Lenau, 1970; Jobson and Sayre, 1970; Parker, 1978; Van Rijn, 1986). For the case of a fully developed flow field,

$$U \frac{\partial C}{\partial x} + V \frac{\partial C}{\partial y} = \frac{\partial}{\partial y} \left(E_P \frac{\partial C}{\partial y} \right) \tag{27}$$

Here, U is the velocity in the x direction, and V is a constant equal to the negative of the free-fall velocity. There seems to be some differences on how to formulate the boundary conditions. Most authors use Eq. 26 as the condition at the top boundary. Consistent with this approach, Eq. 26 should be set equal to the net deposition rate (the difference between the deposition rate of particles per unit area, R_D, and the entrainment rate of particles per unit area, R_A) at the bottom boundary. However, because the net deposition rate is not known and because of the nonphysical behavior of Eq. 26 at $y = 0$, the boundary condition is usually fixed empirically at some finite distance from the bottom.

The principal limitation in using Eq. 27 to describe sediment transport is that it is suited for situations for which the entrainment rate of solids from the bottom wall is governed by a gradient transfer process. This is not the case and, as a consequence, boundary conditions are specified empirically. The Lagrangian analysis recognizes, at the outset, that particles do not enter the field by a gradient process; it focuses on the entrainment process and the need to develop a theory for the entrainment rate and for the manner by which particles enter the field.

The intrinsic time dependency of the turbulent diffusivity, of V and of the boundary conditions cannot be properly taken into account by Eq. 27. These effects must be represented in an empirically determined spatial variation of V and E_P.

The Lagrangian analysis suggests that an equation similar to Eq. 27 can be used to describe the behavior of a single source but that it cannot represent the final concentration field.

Results for Single Wall Source of Particles

The behavior of a single source, $S = R_A/H, dx'dt'$, on the bottom wall was calculated by finite difference methods from Eqs. 3 and 13, using boundary conditions 14 and 15. Uniform grid spacings of Δy and Δt were used. At $t = t'$ the value of

C at the first grid point from the wall was fixed at $S/\Delta y$. The particles were thus spread uniformly over a space Δy from the wall at the time they enter the field. This has the effect of using a finite, rather than an infinitesimal, source at the initial time. For the calculations presented here, $\Delta y/H = 0.01$ and the thickness of the source in wall units is of the order of 10-40. The accuracy of the solution was tested by increasing the number of grid points and choosing the ratio of the time step to the mesh size such that the solutions were stable unchanging with decreasing values.

A fraction of the particles released from the wall source strikes the bottom wall, F_{BB}. These are removed from the field. Another fraction $F_{BT} = 1 - F_{BB}$ strikes the top boundary. As discussed above, these particles are considered to form new sources; however, they maintain the same deterministic velocity they had when they reached the wall. The method for calculating the concentration fields associated with these sources is outlined earlier.

A fraction of the particles originating from these pseudo-sources on the top wall, F_{TB}, will deposit on the bottom wall and be removed from the field. Another fraction $F_{TT} = 1 - F_{TB}$ will return to the top wall to form new sources. This is repeated many times until all of the particles originally released by the line source return to the lower boundary. The computational procedure is detailed by Binder (1991).

As outlined previously, the turbulence properties of the particles were taken equal to the turbulence characteristics of the fluid. The following correlations presented by Vames and Hanratty (1988) were used for the calculations presented here:

$$(\overline{v^2})^{1/2} = 0.9v^* \quad (28)$$

$$\frac{\tau_{Lf} v^*}{H} = 0.046 \quad (29)$$

Three dimensionless groups emerge from these calculations. These are a Froude number defined as:

$$Fr = \frac{v^{*2}}{gH}\left(1 - \frac{\rho}{\rho_P}\right)^{-1} \quad (30)$$

a Reynolds number

$$d_P^+ = \frac{d_P v^*}{\nu} \quad (31)$$

and the dimensionless inertial time constant, $\beta\tau_{LF}$. The product $Fr\beta\tau_{LF}$ varies as (v^*/V_T). For a Stokesian particle,

$$Fr\beta\tau_{LF} = 0.046\,\frac{v^*}{V_T} \quad (32)$$

Figures 2a and 2b present calculations of the concentration at later times that result from a source on the bottom wall at $t' = 0$. The ordinate is the dimensionless concentration given as $C^* = Cv^*/R_A$. The abscissa is the dimensionless distance from the lower wall y/H. The different curves represent different dimensionless times, $t^* = tv^*/H$. Equation 2 shows that $t^* = x\sqrt{f/2}/H$ where f is the friction factor. As a consequence, changes with t^* could be looked upon as changes with distance downstream.

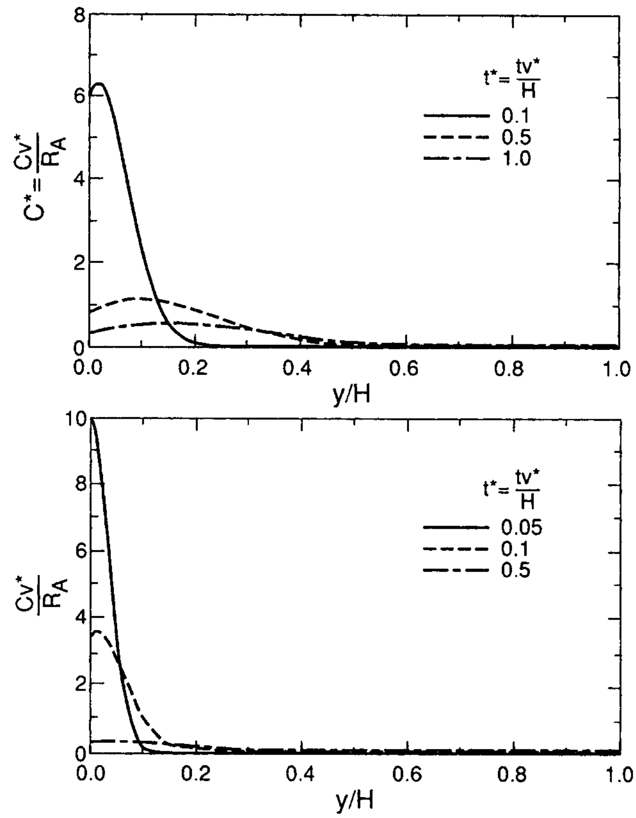

Figure 2. Calculated concentration profiles at different times caused by a single wall source at time zero.

(a) $Fr = 0.01$, $\beta\tau_{LF} = 100$, $d_P^+ = 10$; (b) $Fr = 0.01$, $\beta\tau_{LF} = 10.0$, $d_P^+ = 10$.

Figure 2a was calculated for $Fr = 0.01$, $\beta\tau_{LF} = 100$, and $d_P^+ = 10.0$. The particle is Stokesian so that $V_T/v^* = 0.046$ and $Fr\beta\tau_{LF} = 1$. This corresponds to a case for which the influence of particle settling is small, but not negligible. It is noted that as time increases, the particles spread out and that some reach the top wall. The amount of particles in the field decreases with time because of deposition at the bottom wall. For large enough times, all of the particles deposit and none that entered the field at $t = 0$ remain. For $t \rightarrow 0$, the maximum in the concentration profile is at the wall since $f = 0$. As time proceeds, particles deposit at the wall and the maximum moves away from the wall. The finite concentration gradients at the wall indicate that deposition is being influenced both by diffusion and by gravitational settling. The asymmetry of the concentration profiles is as much a consequence of the location of the sources as of the influence of gravitational settling. Even in the absence of gravitational effects, the particles will tend to congregate at the bottom because they deposit out before they have a chance to reach the top boundary.

The results in Figure 2b were obtained for $Fr = 0.01$, $\beta\tau_{LF} = 10.0$, and $d_P^+ = 10.0$. In this case, $Fr\beta\tau_{LF} = 0.1$ and $V_T/v^* = 0.358$. The particles are non-Stokesian, and the effect of gravitational setting is large. The particles congregate closer to the bottom than for the case in Figure 2a, and a negligible number reaches the top wall.

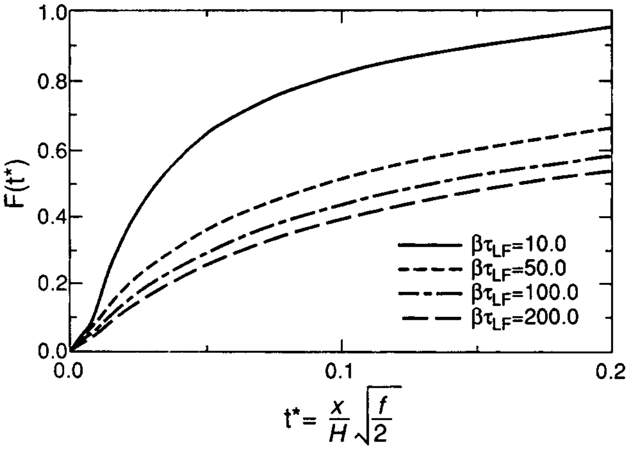

Figure 3. Fraction of the solids from a single wall source that redeposits at later times.

$Fr=0.01$, $d_P^+=10$.

Figure 3 presents a plot of the fraction of the particles, emitted from a line source, that are redeposited on the bottom wall at later times. The calculations were done for $Fr=0.01$ and $d_P^+=10.0$. The different cases considered correspond to values of $Fr\beta\tau_{LF}$ of 0.1, 0.5, 1.0 and 2.0 or V_T/v^* of 0.358, 0.092, 0.046 and 0.023. All of the particles are Stokesian with the exception of the ones characterized by $\beta\tau_{LF}=10$. The deposition is found to decrease monotonically in this range of $\beta\tau_{LF}$ (characteristic of sediment transport in water) with increasing $\beta\tau_{LF}$ or decreasing $V_T|v^*$. The decrease in the rate of deposition with increasing $\beta\tau_{LF}$ (or increasing $Fr\beta\tau_{LF}$) is associated with a decrease in the settling rate due to gravity. The curves in Figure 3 seem to be approaching a lower limit at which deposition is controlled by turbulent diffusion. The concentration profiles in Figure 2a correspond to the $\beta\tau_{LF}=100$ curve in Figure 3. It is a case close to the lower asymptote for which deposition is occurring both by diffusion and settling. The concentration profiles in Figure 2b correspond to the top curve in Figure 3. It is a case for which deposition is controlled by settling.

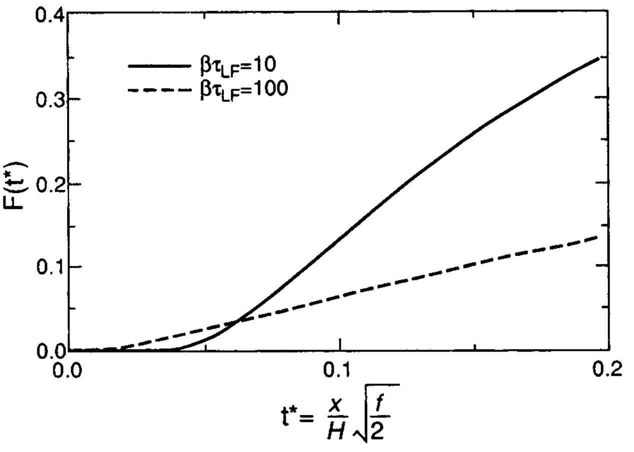

Figure 4. Fraction of solids redeposited at very small times.

$Fr=0.01$, $d_P^+=10.0$.

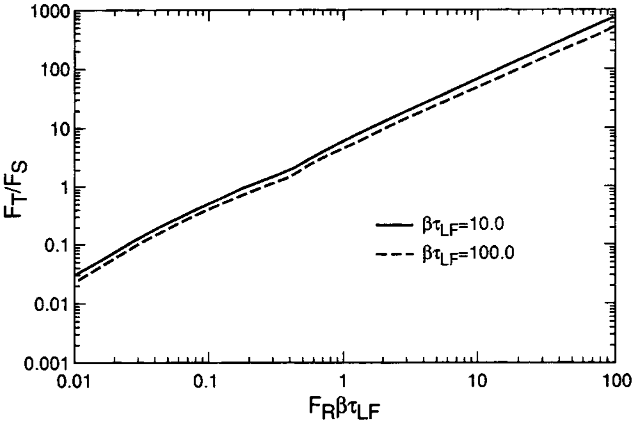

Figure 5. Ratios of the contributions of turbulence and of settling to particle deposition.

For very small $\beta\tau_{LF}$ (such is the characteristic of gas/liquid annular flow or solid/gas flow), an opposite trend would be observed. The deposition rate would decrease with decreasing $\beta\tau_{LF}$ or increasing particle size, because of the inability of the particles to follow the fluid turbulence (Lee et al., 1989a,b; Binder and Hanratty, 1991).

The behavior of the plots of the fraction deposited at small time is shown in Figure 4. Because $f(t)\to 0$ for small time, all of the particles are moving away from the wall for $t\to 0$. As a consequence, the initial deposition rates are quite small.

Figure 5 plots the ratio of the contributions of turbulence, F_T, and setting, F_S, to deposition. Here, from Eq. (6),

$$F_S=\frac{\int_0^\infty (-V)C(0)dt}{\int_0^\infty \left[-V+\sqrt{\frac{2}{\pi}}\,(\overline{v^2})^{1/2}f\right]C(0)dt} \tag{33}$$

and $F_T=1-F_S$. These results are not affected strongly by changes in $\beta\tau_{LF}$. Furthermore, the particle Reynolds number, d_P^+, is important only when the particles are non-Stokesian. The parameter $Fr\beta\tau_{LF}$, therefore, is a good means of characterizing the flow pattern. For $Fr\beta\tau_{LF}>2$, the deposition by turbulence is ten times greater than the deposition by settling. For $Fr\beta\tau_{LF}<0.03$, the deposition by settling is ten times greater that by turbulence. For $0.03<Fr\beta\tau_{LF}<2$, both turbulence and settling are important. The concentration profiles in Figure 2 are for the mixed regime. Figure 2a ($Fr\beta\tau_{LF}=1$) is characterized by $F_T/F_S\cong 6$; Figure 2b ($Fr\beta\tau_{LF}=0.1$) is characterized by $F_T/F_S\cong 0.5$.

The plot of the fraction deposited in Figure 3 for $\beta\tau_{LF}=200$ is just at the beginning of the turbulence-controlled regime, $(F_T/F_S)\cong 10$. It should, therefore, be very close to the lower asymptote. The curves for $\beta\tau_{LF}=10$, for $\beta\tau_{LF}=50$, and for $\beta\tau_{LF}=100$ are in the mixed regime, $(F_T/F_S)\cong 0.5$, 2, 6.

Results for Multiple Wall Sources

The fully developed concentration field may be calculated by adding up the contributions of a large number of infinitesimal wall sources in the following way:

$$C^*\left(\frac{y}{H}\right) = \int_o^t C_{SS}^*(t-t')\,d\left(\frac{t'u^*}{H}\right) \quad (34)$$

where t is allowed to be large enough that $C^*(y|H)$ is not changing with time, and $C_{SS}^*(t-t')$ are the single source solutions in Figures 2a and 2b.

Concentrations calculated in this way are made dimensionless with the bulk concentration C_B and plotted against the logarithm of y/H in Figure 6a for $Fr=0.01$ and $d_P^+=10.0$. These represent a range of $Fr\beta\tau_{LF}$ for which both turbulence and settling are important in controlling deposition. It is noted that the profile becomes increasingly stratified as $F_T|F_S$ changes from 6 to 0.2. Figure 6b gives a pilot of the same results in arithmetic coordinates. The results in Figure 6 cannot be directly interpreted with a time-dependent turbulent diffusion coefficient. They are the result of a superposition of a large number of infinitesimal sources which have been in the field for different lengths of time. The concentration gradients close to the wall, however, are influenced more by sources which have been in the field shorter times. Concentration gradients far from the wall are influenced by particles that have been in the field long times. (See Hanratty, 1958, and Eckelman and Hanratty, 1972, for a discussion of these aspects of the results.)

In the region of $0.2<y|H<0.6$, the turbulent diffusivity is approximately constant and the O'Brien relation (Eq. 26) gives:

$$\frac{d\ln C}{dx} = -\frac{V_T}{E_P} \quad (35)$$

Values of E_P were calculated from Eq. 35 and the plots in Figure 6a. These are presented in Figure 7 as the ratio of $E_P|\overline{v^2}\tau_{LF}$. It is noted that the use of Eq. 34 produces ratios of the particle diffusivity to fluid diffusivity different from unity, as has been noted by a number of researchers (Karabelas, 1977; Hunt, 1964, 1969). One possible explanation is that the effective settling velocity is smaller than V_T because the particles have not been in the field long enough to reach their free-fall velocity. The use of a value of the settling velocity in Eq. 35 that is too large would produce a larger E_P and could account for ratios of the particle and fluid diffusivity which are greater than unity. However, it is more difficult to explain values of this ratio less than unity.

A more likely interpretation of Figure 7 is that the basic assumption of Eq. 26 is incorrect, in that there is a flux of particles toward the bottom wall as a result of settling and diffusion. A dynamic equilibrium should exist between the particles in the fluid and particles at the wall whereby the deposition rate is balanced by an entrainment rate. Unlike cases involving molecular transfer, the mechanism for transport of particles away from a wall is quite different from that for the transport of molecular species away from a wall. As a consequence, it is difficult to interpret particle concentration profiles with concepts derived from molecular transport. Along these lines, it is interesting to note from Figure 7 that $(E_P|\overline{v^2}\tau_{LF})=1$ at $Fr\beta\tau_{LF}\cong 0.2$ for which $F_T|F_S\cong 1$. Values of $E_P|\overline{v^2}\tau_{LF}>1$ correspond to cases for which Figure 5 indicates that settling is more important than turbulence in the deposition process. Conversely, values less than unity correspond to $F_T>F_S$.

From the concentration profiles in Figure 6, a depth, δ, of

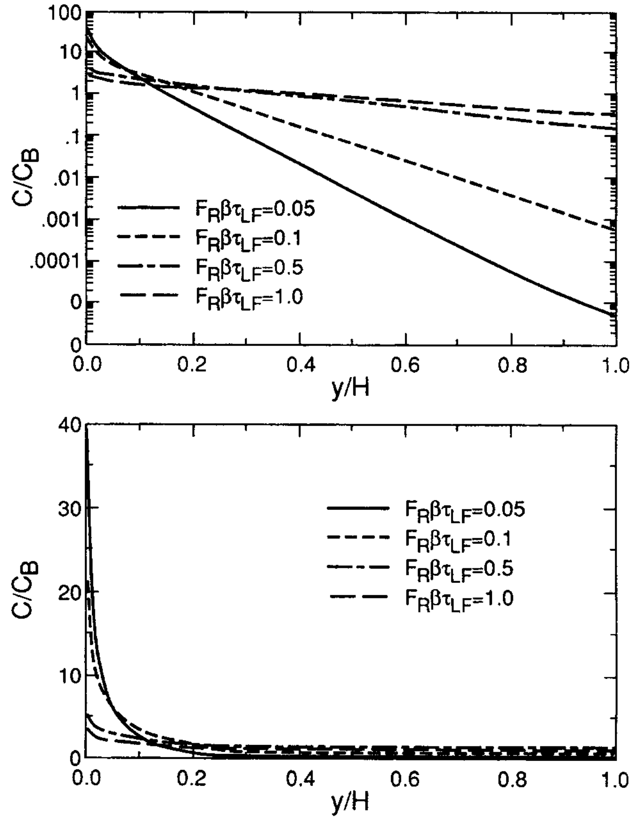

Figure 6. Calculated fully developed concentration profiles obtained by summing the contributions from multiple sources.

(a) semilog coordinates; (b) arithmetic coordinates.

the suspended particles can be calculated if $\delta|H$ is defined as the location where $C|C$ wall $=0.01$. In this way, values of $\delta|H=0.22$ and $\delta|H=0.40$ are calculated for $Fr\beta\tau_{LF}=0.05, 0.1$. Using this criterion, the suspended solids fill the entire channel for $Fr\beta\tau_{LF}=0.5$, 1.0. This would suggest that there is a fuzzy liquid slurry interface for small $Fr\beta\tau_{LF}$ and that this interface ceases to exist at a value of $Fr\beta\tau_{LF}$ between 0.1 and 0.5.

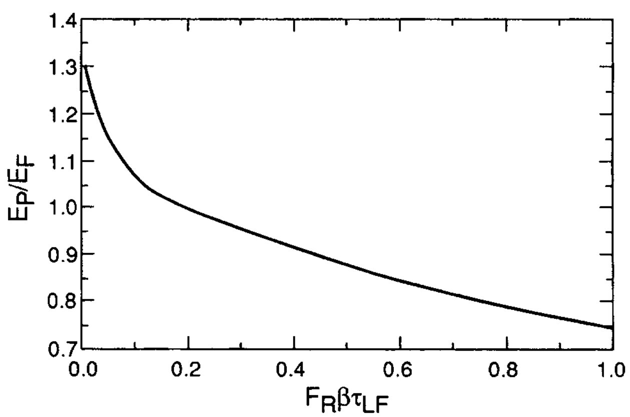

Figure 7. Particle Eulerian diffusion coefficients calculated from the slope of the concentration profiles in Figure 6 between $y/H=0.2$ and $y/H=0.8$.

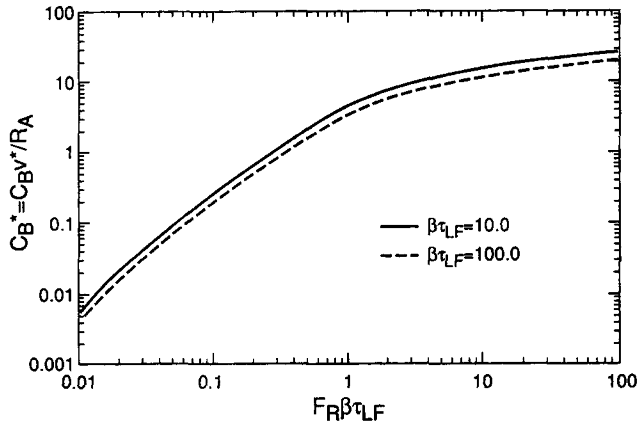

Figure 8. Bulk concentration (or transported bed load) as a function of $Fr\beta\tau_{LF}$.

The quantity of solids transported in the fluid, the suspended load, can be represented by the bulk concentration C_B. Calculated values of $C_B|R_Av^*$ are plotted in Figure 8. Again, it is noted that the suspended load is primarily a function of $Fr\beta\tau_{LF}$.

Discussion

Sediment transport in liquids is considered for dilute enough concentrations that particle-particle interactions and the influence of the particles on the turbulence can be neglected. It is argued that a physical understanding of the process is better achieved by considering the concentration field as resulting from a distribution of sources. The advantages of this approach over a Eulerian representation were discussed previously, which are similar to what is used to describe transport of molecular species.

The calculations are simplified greatly by assuming a homogeneous field: that is, the large changes in the velocity field and the turbulence close to a boundary need not be considered. This is justified because sediment transport is usually characterized by conditions such that the particle stopping distances are greater than the thickness of the viscous wall region. As a consequence, particles moving toward the boundaries are not influenced by turbulence in the viscous wall region. Furthermore, it is argued that the entrainment process is such that particles may be considered to be fully entrained in the turbulence at a location beyond the edge of the viscous wall region.

The application of this analysis to describe the fully developed concentration field of uniformly sized particles shows that the behavior is controlled primarily by $Fr\beta\tau_{LF}$ or $V_T|v^*$. Two methods are explored for characterizing the flow pattern. One of these considers the relative roles of turbulence and settling in depositing particles. The other considers the location of a slurry-liquid interface defined (arbitrarily) as the distance from the lower boundary where the concentration is 1% of the concentration at the wall.

For $Fr\beta\tau_{LF}=0.2$, the relative contributions of turbulence and settling to deposition are equal, $F_T|F_S=1$. For $Fr\beta\tau_{LF}>2$, $F_T|F_S>10$; for $Fr\beta\tau_{LF}<0.03$, $F_T|F_S<0.1$. The level of the suspended solids increases with increasing $Fr\beta\tau_{LF}$. The suspension fills the whole channel cross section at a value of $Fr\beta\tau_{LF}$ close to 0.2.

The suspended load is defined in terms of a bulk-averaged concentration, C_B. For a given $Fr\beta\tau_{LF}$, it is found that $C_Bv^*|R_A$ has a fixed value. If the total flow of solids is less than this value, it would be expected that no settled bed would exist at the bottom wall. The suspended load would then be calculated simply from the solids flowing into the system.

Accuracy of the Analysis

This article cannot be concluded without emphasizing a number of assumptions that affect the accuracy of the calculations. The goal has been to obtain a first-order representation of sediment transport in liquids. It is worthwhile, therefore, to use the results of this analysis as a means to examine errors associated with different simplifications of the problem.

As mentioned earlier, the assumption of a homogeneous field could be avoided by analyzing the behavior of a single source by random flight calculations of a large enough number of particle trajectories to obtain good statistics. However, methods for introducing the effects of turbulence nonhomogeneities are untested. At present, it appears that this type of analysis would be most advantageously used to evaluate particle dispersion in a homogeneous turbulence and a nonuniform average velocity field.

The present analysis focuses on the importance of a physical understanding of how the particles enter the field. It requires knowledge regarding the initial velocities, U_o and V_o, and the rate of entrainment, R_A. Contributions in this area have been made by Francis (1973) and by Nalpanis (1985a,b), but more needs to be done. At present, however, it would seem important to examine how the calculations would be influenced by the choice of V_o and U_o. The assumption that $V_o\cong(\overline{v^2})^{1/2}$ seems consistent with the results quoted by Hunt and Nalpanis (1985) that $V_o\cong 2v^*$, since $(\overline{v^2})^{1/2}=0.9v^*$. However, it is to be noted that our choice of V_o is about 50% of the value recommended by Hunt and Nalpanis.

The main influence of U_o is in the calculation of $U(t)$ along the deterministic trajectory. This, in turn, affects the calculation of $V(t)$ because of the influence of $U(t)$ on the drag. For a Stokesian drag the calculation is independent of the choice of U_o. However, if nonlinear drag effects are important, no slip in the flow direction $[U(t)=U^F]$ assumed here could introduce errors, particularly in the saltation region. For example, Hunt and Nalpanis (1985) considered solid particles in air and showed for a square resistance law that the maximum height of the deterministic trajectory is reduced by a factor of two from what would be obtained with a linear flow.

Finally, it should be pointed out that the deterministic equation of motion of the particles is simplified. Time-averaged velocity gradients in the fluid can introduce lift effects (Rubinow and Keller, 1961; Saffman, 1965; Thomas et al., 1984; White, 1982) that were not considered and the drag coefficient can depend on the turbulent velocity fluctuations as well as on the mean slip velocity (Mei, 1990).

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Notation

A_p = projected area of drop
C = concentration
C^* = Cv^*/R_A
C_B = bulk concentration
$C_{SS}(t-t')$ = solution for concentration field at time t for a single source or a source-sink pair at time t'
d_P = particle diameter
d_P^+ = $d_P v^*/\nu$
D = molecular diffusion coefficient
E_P = Eulerian turbulent diffusion coefficient for a particle
f = fraction of particles from a single source moving toward the wall where the source is located; the friction factor
f_D = drag coefficient
Fr = Froude number = $v^{*2}/gH\,[1-(\rho/\rho_p)]^{-1}$
F_{BT} = fraction of particles from a source on the bottom wall hitting the top wall
F_{TB} = fraction of particles from a source on the top wall hitting the bottom wall
F_S = contribution of settling to deposition
F_T = contribution of turbulent to deposition
g = acceleration of gravity
H = channel height
L = characteristic length
m_P = mass of a particle
N_i = particle flux
$|Q_R|$ = absolute relative velocity between the particle and fluid
R_A = entrainment rate
R_D = deposition rate
R_{DH} = deposition rate to the top wall
R_{DO} = deposition rate to the bottom wall
R_{DO}^S = contribution to R_{DO} due to gravitational settling
R_{DO}^T = turbulence contribution to R_{DO}
R_L = Lagrangian correlation coefficient
S = ratio of particle velocity to the fluid velocity; source strength
t = time
t' = time at which a source admits material to the field
t^* = tv^*/H
t_S = time at which solid particles from a source on the bottom wall reach the top wall
u = turbulent velocity component of the particle in the x direction
u_o = initial turbulent velocity component of the particle in the x direction
U = particle velocity in the flow direction
U_F = fluid velocity in the x direction
U_o = initial velocity in the x direction with which a particle enters the field
U_R = x component of the relative velocity between the particle and the fluid
U = characteristic velocity
v = turbulent velocity component of the particle in the y direction
v^* = friction velocity
v_o = initial turbulent velocity component of the particle in the y direction
$\overline{v^2}$ = average value of v^2
V = particle velocity in a direction perpendicular to a wall
V_H = velocity with which particles enter the field from top boundary
V_o = initial velocity in the y direction of particles entering the field
V_R = y component of the relative velocity between the particle and fluid
V_T = terminal velocity of a particle
x = coordinate in the flow direction
$\vec{x}'$ = location of a source
y = coordinate perpendicular to the wall
Y = y coordinate of a diffusing particle that was at $y=0$ at $t=0$
$\overline{Y^2}$ = average value of Y^2 for a large number of particles

Greek letters

β = reciprocal time constant for a particle defined by Eq. 1
ϵ = Lagrangian diffusivity of the fluid
η = $y-\int V dt$
ν = kinematic viscosity
ρ = fluid density
ρ_P = particle density
τ_{LF} = Lagrangian time constant for the fluid
τ_P = inertial time constant for the particle $=\beta^{-1}$
τ_P^+ = $v^{*2}\tau_P/\nu$

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